Impact of data break on the uncertainty of GNSS site velocity estimate

Lei Wang[1*], Thomas A. Herring[1]

[1]Department of Earth, Atmospheric and Planetary Science, *Massachusetts Institute of Technology*, US

*Now at Department of Civil, Environmental and Geodetic Engineering, *The Ohio State University*, US

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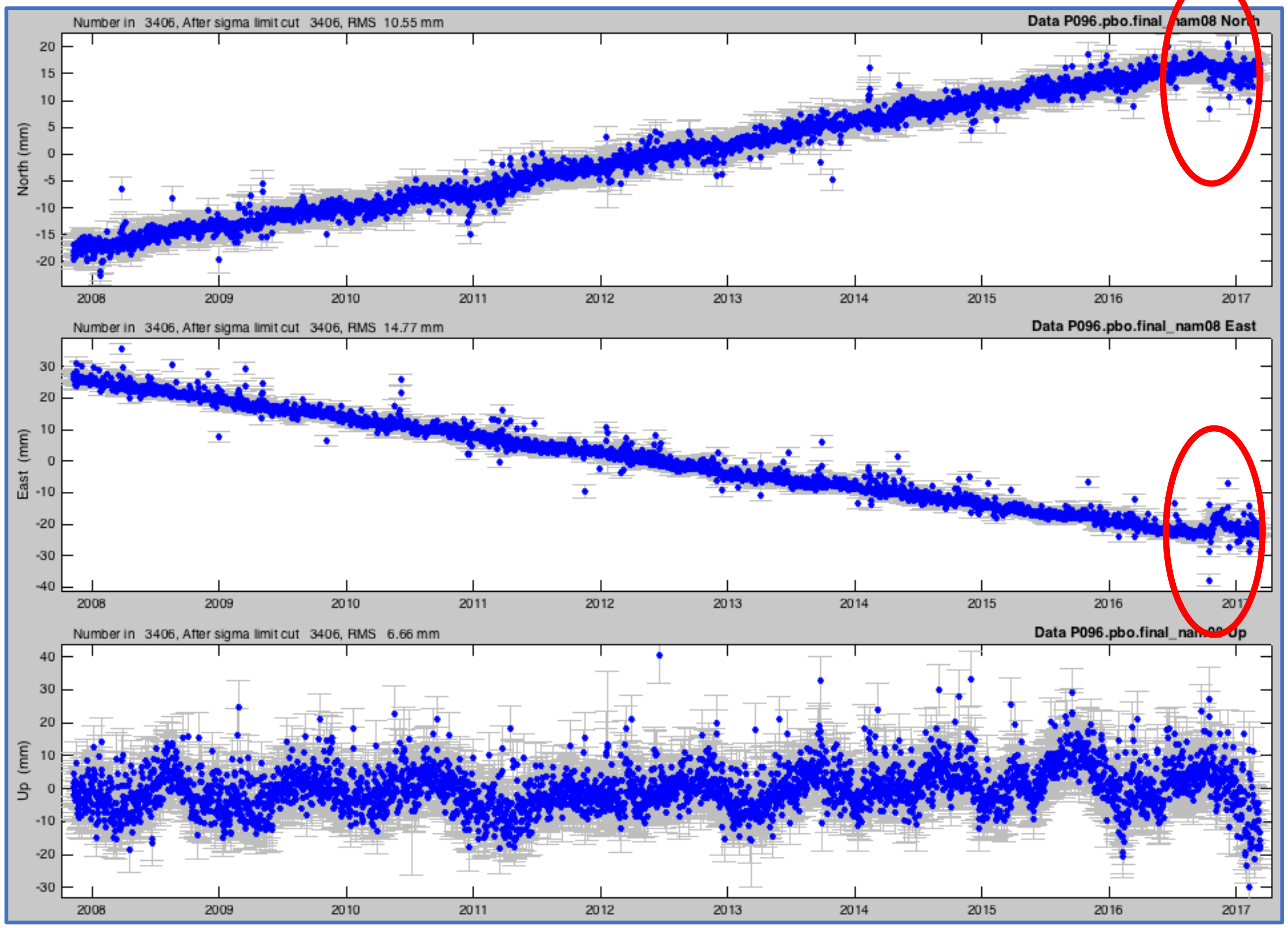
Introduction

Accurate estimation of GNSS site velocity and its uncertainty is essential for many geodetic and geophysical applications:

- Reference frame realization
- Plate tectonic deformation
- Glacial isostatic adjustment
- Crustal loading deformation



Data breaks commonly exist due to either natural or artificial causes, such as :

- Earthquake
- Environmental changes
- Equipment change
- Changes in processing strategy,
- Human error

Site velocity estimate and its uncertainty

$$y_t = c + r \cdot t + \sum_{i=1}^{K} b_i \cdot p_i + a_1 \cdot cos2\pi f_0 t + a_2 \cdot sin2\pi f_0 t + e_t,$$ where $p_i = \begin{cases} 0 & \text{if } t < t_i \\ 1 & \text{if } t \geq t_i \end{cases}$, Heaviside step function

Given the covariance matrix of the observational noise, $\mathbf{C_e}$, the least square estimate for the unknown vector

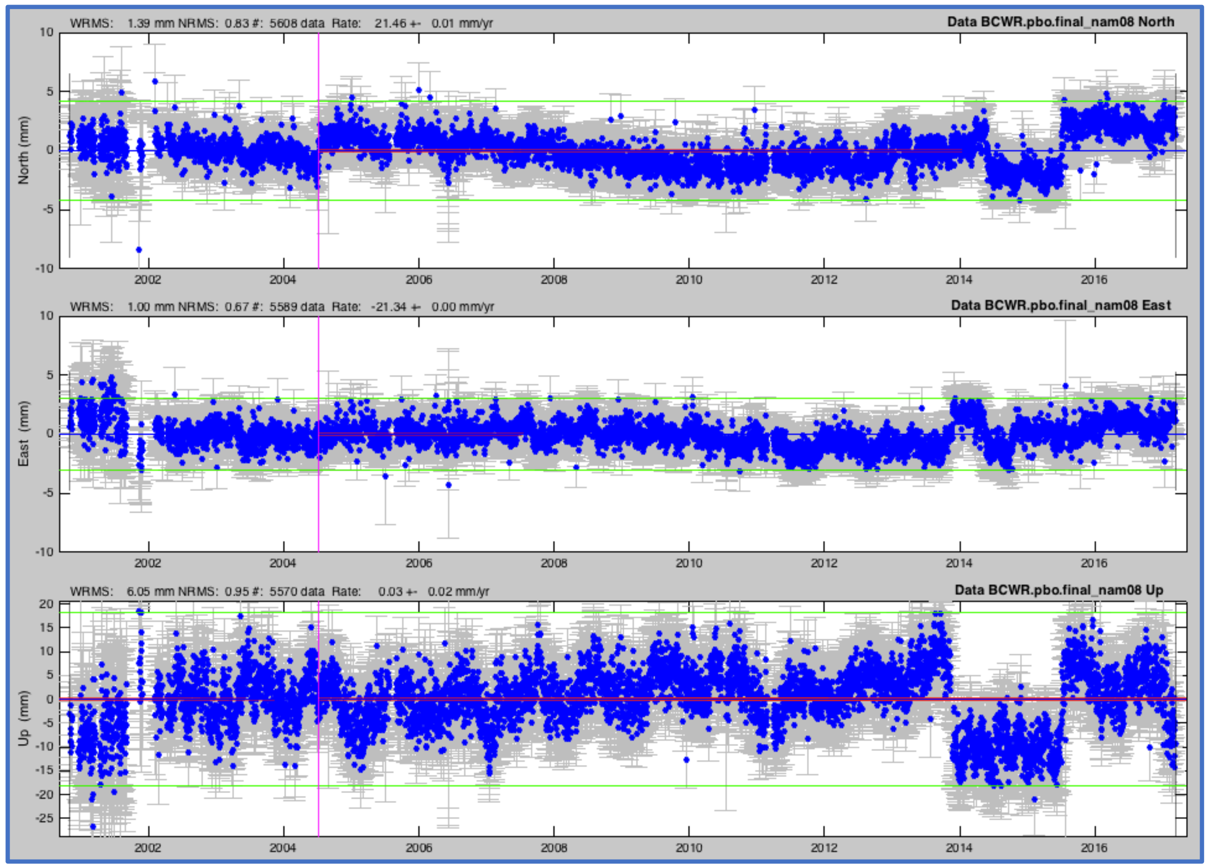
$$\hat{\boldsymbol{x}} = \left[\hat{c}, \hat{r}, \hat{a}_1, \hat{a}_2, \hat{b}_1, \cdots, \hat{b}_K\right]^T$$

is

$$\hat{\mathbf{x}} = \left(\mathbf{A^T C_e^{-1} A}\right)^{-1} \mathbf{A^T C_e^{-1} y} \,,$$

and the posterior covariance matrix for the estimate is

$$\mathbf{C_{\hat{X}}} = \left(\mathbf{A^T C_e^{-1} A}\right)^{-1} .$$



Noise in GPS position time series

Power-law noise:

$$P_e(f) = P_0\left(\frac{f}{f_0}\right)^{\kappa}$$

where

P_0 and f_0 : normalization constants

κ is spectral index:

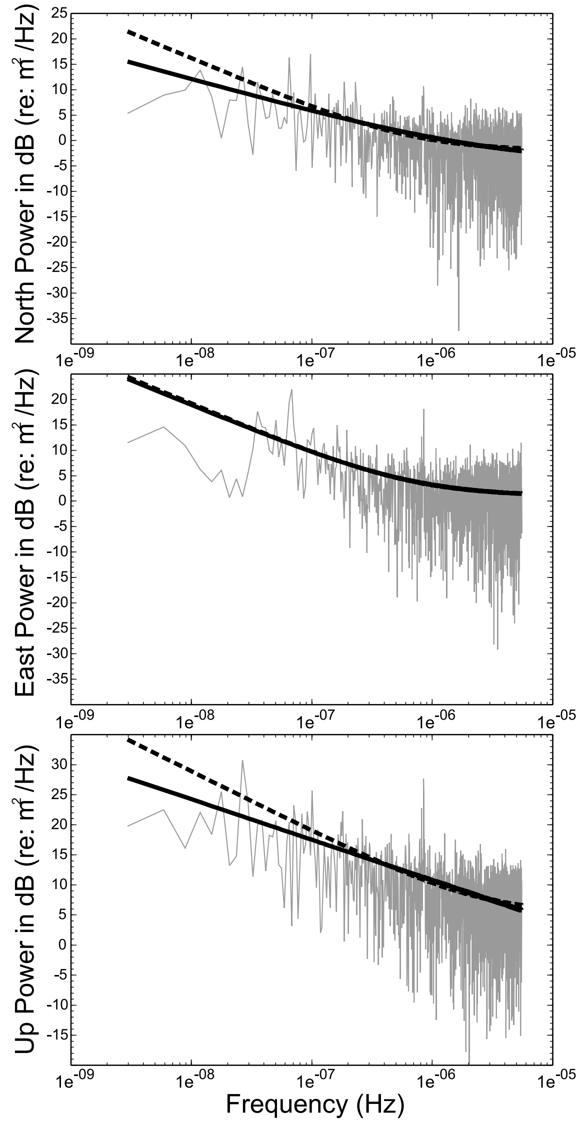
$\kappa = 0$: White noise

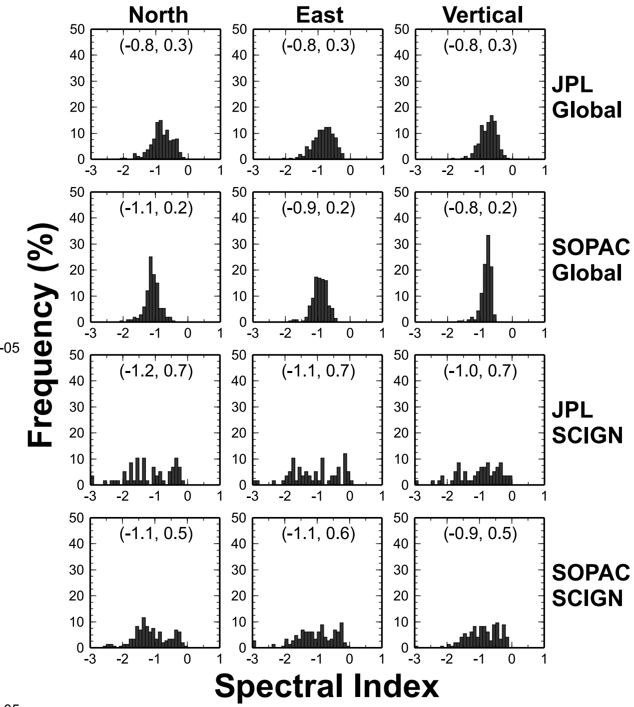
κ= -1: Flicker noise

κ= -2: Random walk

-3 < κ < -1: Fractal random walk

κ > -1 : Fractal white noise





Williams et al. [2004]

Noise covariance matrix

$$\mathbf{C}_{\hat{\mathbf{X}}} = \left(\mathbf{A}^{\mathbf{T}}\mathbf{C}_{\mathbf{e}}^{-1}\mathbf{A}\right)^{-1}$$

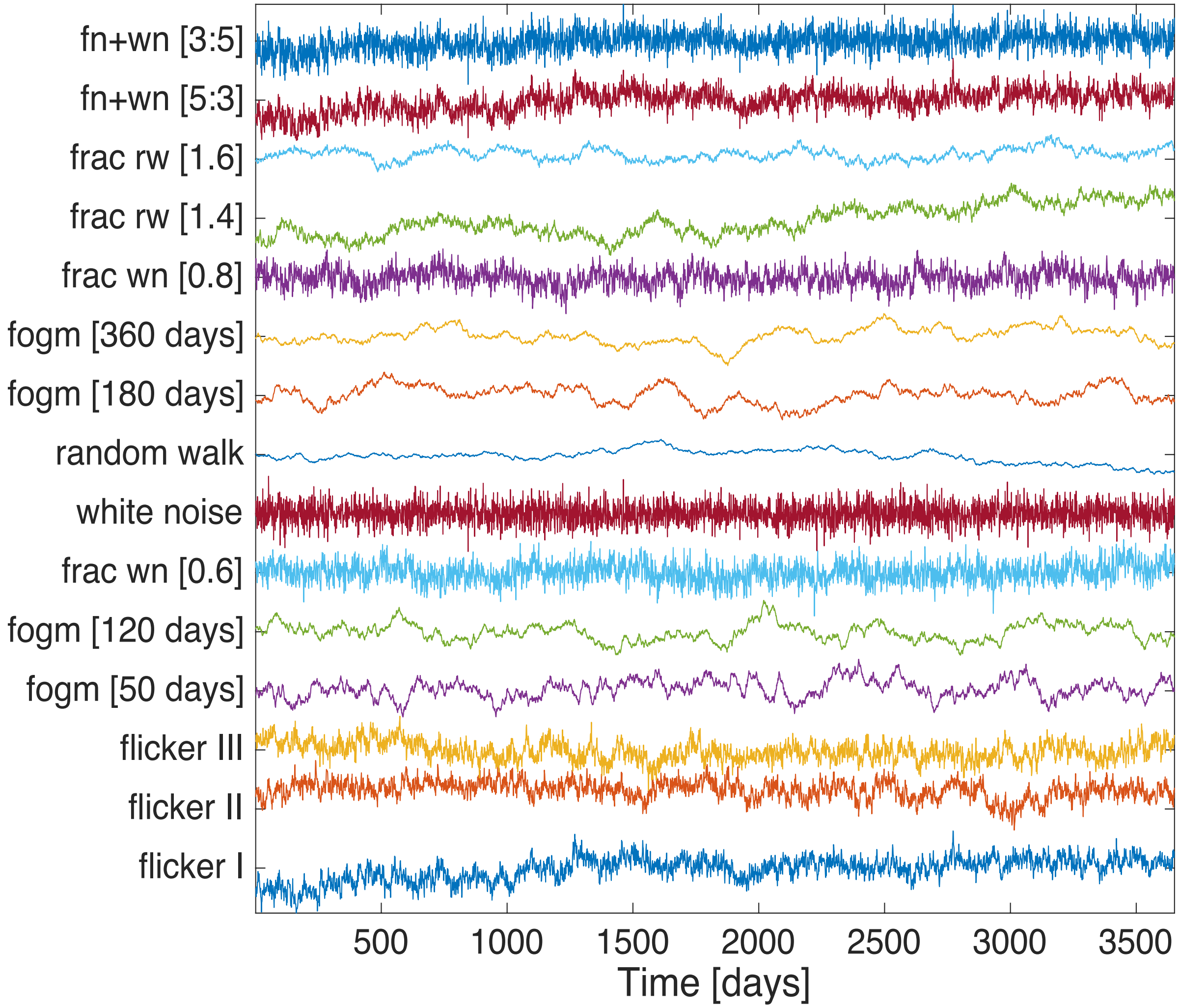
Power law spectrum: $P_e(f) = P_0\left(\frac{f}{f_0}\right)^{\kappa}$

Auto-correlation function $\varphi_e(\tau)$

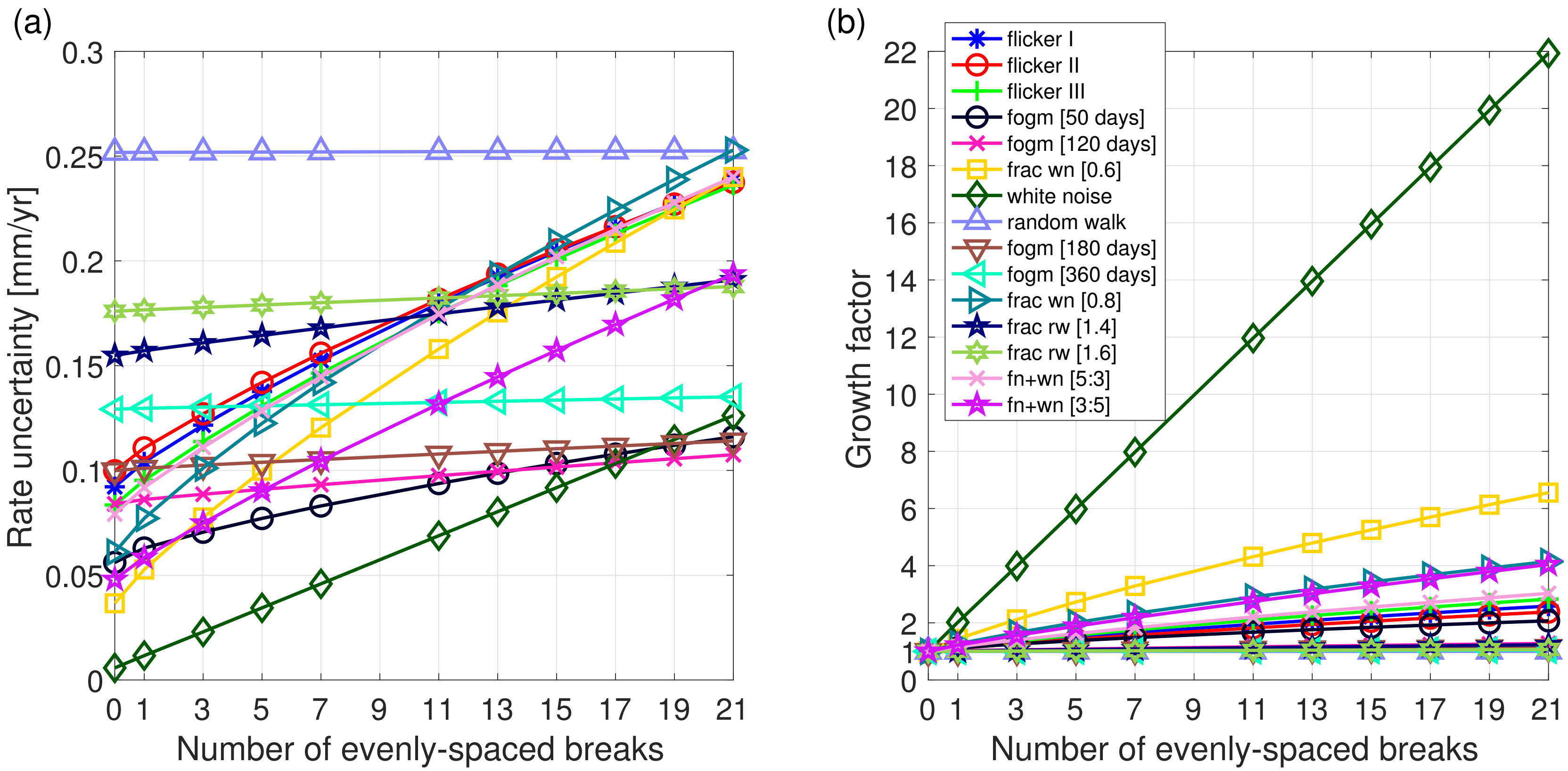
Initial covariance matrix $\mathbf{C}_{\mathbf{e}}^{\mathbf{0}}$

Synthesize 400 decade-long noise time series with daily sampling rate

Scale $\mathbf{C}_{\mathbf{e}}^{\mathbf{0}}$ so that the mean RMS of the 400 noise time series is 1 mm.



Sensitivity of rate uncertainty to data breaks



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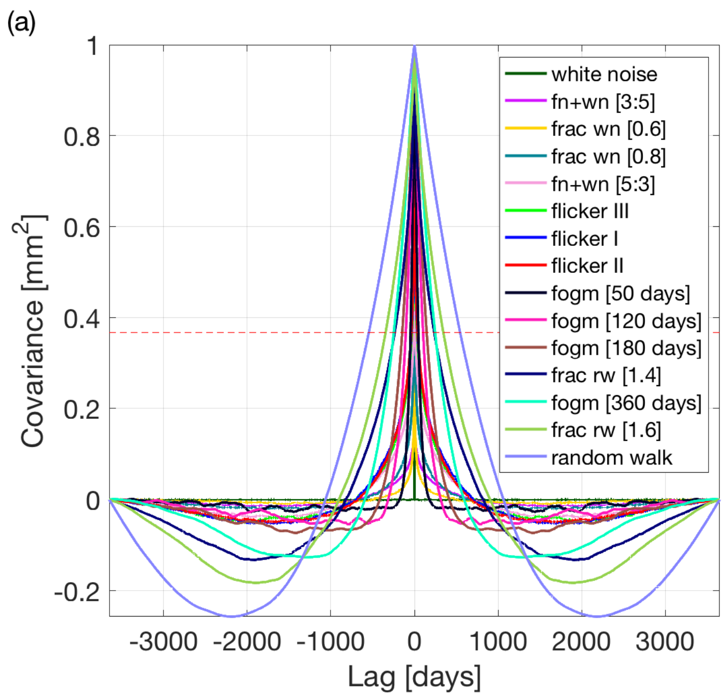
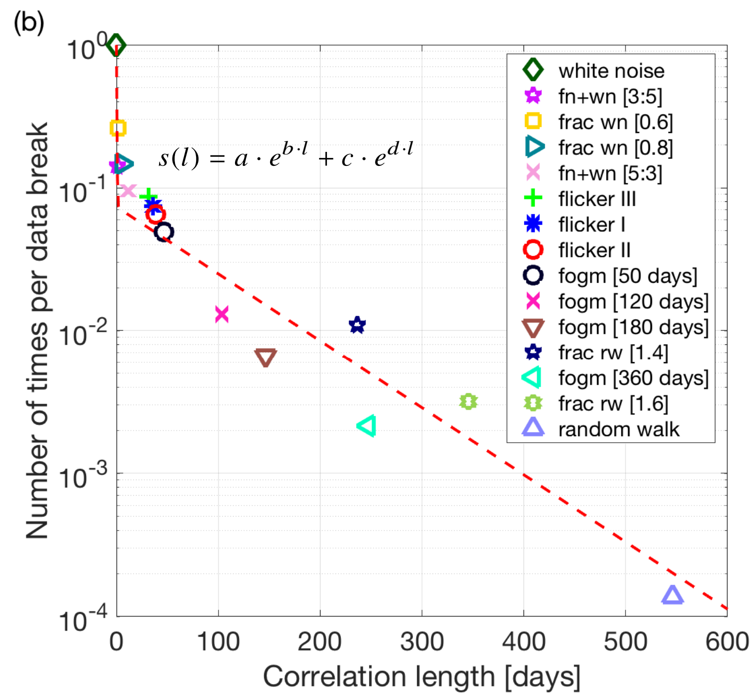
Sensitivity of rate uncertainty to data breaks





$$c(m) = \begin{cases} \sum_{n=0}^{N-m-1} \left(e_{n+m} - \frac{1}{N} \sum_{i=0}^{N-1} e_i \right) \cdot \left(e_n - \frac{1}{N} \sum_{i=0}^{N-1} e_i \right), & \text{if } m \geq 0 \\ c(-m), & \text{if } m < 0. \end{cases}$$

$$s(l) = 0.923 \cdot e^{-3.061 \cdot l} + 0.074 \cdot e^{-0.011 \cdot l}$$

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Conclusion

- The estimated site velocity becomes more uncertain as the number of data breaks increases.
- How fast the uncertainty increase depends on the degree of temporal correlation of the observational error. The sensitivity of rate uncertainty to data breaks decrease rapidly as the degree of correlation increases.
- The simple white noise assumption overestimate the sensitivity of rate uncertainty to data breaks.
- For the mixture of flicker noise and white noise, which best describe the noise in most GPS position time series, doubling the number of data breaks from 1 to 2 only inflates the rate uncertainty by 11%.
- The existence or potential future growth of data breaks is not a major factor undermining the stability of reference frame realization, once the temporal correlation of the error is properly accounted for.
- The necessary equipment replacement, which is critical to improve GNSS data quality, should be done without hesitation.