INTRODUCTION Some GNSS systems, such as GPS and Galileo, have similar frequencies, i.e. the L1-E1 (1575.42 MHz) and the L5-E5a (1176.45 MHz) frequencies. These overlapped frequencies are beneficial for mixed RTK positioning, relying on integer ambiguity resolution. Instead of forming double-differenced phase and code observations for each GNSS separately (thereby choosing a pivot satellite for each system), with the overlapping frequencies it is possible to form the double differences with respect to the pivot satellite of just one system. As a consequence, additional parameters need to be estimated in the system of mixed observation equations: these are the differential Inter-System/Signal Biases (ISBs), accounting for the differential receiver hardware delay difference between the different systems and signals phase and code observations on overlapping frequencies. This contribution presents results of differential ISBs estimated from GPS plus GIOVE data tracked in Western Australia.

DIFFERENTIAL GPS+GALILEO INTER-SYSTEM BIASES

Traditional single and double differencing

Traditionally, short-baseline RTK positioning applications are based on first forming single-differenced (SD) phase and code observation equations, eliminating satellite-dependent biases. Assume a rover receiver 2 with respect to a reference receiver 1, then the following SD observation equations can be set up, for satellite $i$ and receiver $j$ for a multi-GNSS receiver tracking GPS as well as Galileo measurements:

\[
\begin{align*}
\text{GPS SD:} \quad & E(p_{i0,j}) = \rho_{i0,j}^r + \rho_{i0,j}^s + \lambda M_{i0,j}^s \\
\text{Galileo SD:} \quad & E(p_{i0,j}^G) = \rho_{i0,j}^{rG} + \rho_{i0,j}^{sG} + \lambda M_{i0,j}^{sG}
\end{align*}
\]

where $E(t)$ denotes the expectation, $\lambda_i$ the wavelength, $m_i$ the number of GPS satellites and $m_j$ the number of Galileo satellites. Furthermore:

- $\rho_i^r$: SD code observable
- $M_{ij}^s$: based differential phase observable $= \Delta \phi_{ij}^s = \phi_{ij}^s - \phi_{12}^s$
- $d_{ii}$: SD receiver clock error $M_{ij}^{sG}$
- $S$:
  - SD integer ambiguity
- $q$
  - SD non-integer ambiguity

The (biased) receiver hardware delays are different for both GPS and Galileo, despite that the signals are tracked inside one receiver [1]. For both GNSSs a pivot satellite is used, i.e. 1 for GPS and 11 for Galileo, as to define the double-differenced (DD) ambiguities. Instead of solving the SD observation equations, DD observation equations can be formed by taking the difference of a SD observation with the SD of the pivot satellite, eliminating the receiver hardware biases:

\[
\begin{align*}
\text{GPS DD:} \quad & E(p_{i0,j}^G) = \rho_{i0,j}^r + \lambda M_{i0,j}^s \\
\text{Galileo DD:} \quad & E(p_{i0,j}^{rG}) = \rho_{i0,j}^{rG} + \lambda M_{i0,j}^{sG}
\end{align*}
\]

The phase ISB parameter is only estimable as biased by the integer ambiguity between GPS and Galileo pivot satellites (the so-called inter-system ambiguity).

Galileo double differencing with respect to GPS: ISB estimation

Since GPS and Galileo phases have the same frequencies, i.e. GPS L1 and Galileo E1 (1575.42 MHz) and GPS L5 and Galileo E5a (1176.45 MHz), we can also form Galileo double differences relative to the pivot satellite of GPS [2]:

\[
\begin{align*}
\text{GPS DD:} \quad & E(p_{i0,j}^G) = \rho_{i0,j}^r + \lambda M_{i0,j}^s \\
\text{Galileo DD:} \quad & E(p_{i0,j}^{rG}) = \rho_{i0,j}^{rG} + \lambda M_{i0,j}^{sG}
\end{align*}
\]

where we have one Galileo double-difference more for phase and code, but also one parameter more for both phase and code: the differential Inter-System Biases (ISB):

\[
\begin{align*}
\delta_{ii}^s &= d_{ii}^s - \Delta \phi_{12}^s + \lambda z_{12} \\
\delta_{ii}^{sG} &= d_{ii}^{sG} - \Delta \phi_{12}^{sG}
\end{align*}
\]

The phase ISB parameter is only estimable as biased by the integer ambiguity between GPS and Galileo pivot satellites (the so-called inter-system ambiguity).

Galileo double differencing with respect to GPS: ISB correction

If the differential phase and code ISBs are known, we can correct the Galileo phase and code observation equations to improve the strength of the model and consequently ambiguity resolution and RTK positioning. Let us denote the phase and code ISB corrections as:

\[
\begin{align*}
\delta_{ii}^s &= -d_{ii}^s + \lambda z_{12} \\
\delta_{ii}^{sG} &= -d_{ii}^{sG}
\end{align*}
\]

Due to the corrections, the Galileo single differences become parameterized into the (biased) receiver clocks of GPS, that thus can be eliminated by (double) differencing with respect to the GPS pivot satellite.

\[
\begin{align*}
\text{ISB-corr. Galileo SD:} \quad & E(p_{i0,j}^G) = \rho_{i0,j}^{rG} + \lambda M_{i0,j}^{sG}
\text{ISB-corr. Galileo DD:} \quad & E(p_{i0,j}^{rG}) = \rho_{i0,j}^{rG} + \lambda M_{i0,j}^{sG}
\end{align*}
\]

Now both DD observables as well as the estimable integer ambiguity for Galileo are relative to the pivot satellite of GPS. The presence of $z_{12}$ is not an issue since it is integer and subtracted for every Galileo satellite. Thus, in this way the Galileo double-differences are fully interoperable with those of GPS.