Thermal Re-Radiation Acceleration in the GNSS Orbit Modelling Based on Galileo Clock Parameters

Drazen Svehla

M. Rothacher, L. Cacciapuoti

1ETH Zurich, Institute of Geodesy and Photogrammetry, Switzerland
2ESA/ESTEC, Directorate of Science and Robotic Exploration, Noordwijk, Netherlands
Galileo Clock Parameters in the Sun-fixed Frame (daily time drift/bias removed)

\[ \Delta r = A \cdot \cos \beta \cos \Delta u \]

\[ \delta_{clk} = \Delta r = -A \cdot \cos \varepsilon \]
Comparison of SLR and Galileo Clock

STD(MGEX CLOCK) = 20.7 mm
STD(Simulated CLOCK) = 15.5 mm
STD(Orbit) = 14 mm
STD(SLR) = 25.3 mm

Very good agreement between SLR and Galileo Clock!

Galileo clock is showing smaller standard deviation compared to SLR!
(MGEX and ground test)
Relation Between Clock, SLR Bias and Orbit Translation (geocenter)

Galileo Clock

Setting Sun Galileo E11 (AIUB)

Sun Elevation in [°]

Δu angle

Orbit Translation

ΔZ_{geocenter} \sim \sin \beta

SLR Residuals = "observed" - "calculated"

\[ \Delta r = A \cdot \cos \beta \cos \Delta u \]
\[ \delta_{clk} = \Delta r = -A \cdot \cos \varepsilon \]

SLR Bias:

\[ \bar{\delta}_{SLR} = \bar{\delta}_{const} + \sum \frac{n_i \cdot A \cos \varepsilon_i}{\sum n_i} = -2.4 \text{ cm} - 4.1 \text{ cm} = -6.5 \text{ cm} \]

albedo? (wrong sign! increases the SLR bias)

antenna thrust? (wrong sign! increases the SLR bias)
Thermal Re-Radiation Acceleration

Stefan-Bolzman Law:

\[ q_{\text{therm}} = q_{\text{front}} - q_{\text{rear}} = 5.67 \times 10^{-8} \left( \varepsilon_{\text{front}} T_{\text{front}}^4 - \varepsilon_{\text{rear}} T_{\text{rear}}^4 \right) \quad \left[ \text{W/m}^2 \right] \]

Stefan-Boltzmann constant

Acceleration induced by the thermal radiation emission

\[ a_{\text{therm}} = -\frac{2}{3} C_{\text{therm}} \frac{A}{m} \left( \varepsilon_{\text{front}} T_{\text{front}}^4 - \varepsilon_{\text{rear}} T_{\text{rear}}^4 \right) \mathbf{n} \]

Surface temperature (front, rear)

Flat Surface

Diffuse Emission

Solar Panels

Satellite Body
Thermal Re-Radiation Effect and Thermal Inertia (Yarkovsky Effect)

Solar Panel:

Satellite Body:

Thermal Flux Balance

\[ q^{in} = \alpha \cdot q^{Sun} = q^{out}_{front} + q^{out}_{rear} \]

Thermal re-radiation acceleration

\[ \mathbf{a}_{panel} = -\frac{2 \times 5.67 \times 10^{-8}}{3} \frac{A}{c \cdot m} \left( \varepsilon_{front} T_{front}^4 - \varepsilon_{rear} T_{rear}^4 \right) \mathbf{n} \]

\[ \mathbf{a}_{body} = -\frac{2 \times 5.67 \times 10^{-8}}{3} \frac{A}{c \cdot m} \varepsilon_{body} T_{body}^4 \mathbf{n} \]
Thermal Re-Radiation Effect and Thermal Inertia (Yarkovsky Effect)

Solar radiation (SRP)  
Thermal (infrared) re-radiation acceleration

Δu = 0° PM
Δu = 180° PM
Δu = 90° PM
Δu = 270° PM

Orbit
Midnight
Noon

Estimated thermal inertia = 4.7 min

Hill Equations:
exact solution for the periodic (orbit) acceleration

\[ \Delta a_i = A_i \cos nt + B_i \sin nt + C_i \quad i = \begin{cases} \text{radial} \\ \text{along} \\ \text{across} \end{cases} \]

\[ \Delta x_i = f(A_i, B_i, C_i, n) \quad i = \begin{cases} \text{radial} \\ \text{along} \\ \text{across} \end{cases} \]

(Colombo, 1989)

SLR Residuals (1/2 draconic year)

- SLR: Rising Sun Galileo E11 (MGEX AIUB)

Mean=0.8 cm  σ=10.1 cm

Effect of the Yarkovsky thermal re-radiation

Clock (radial) residual in [m]

SLR Residuals [m]

Δu angle
Allan Deviation

Galileo E11 Frequency Stability

- time drift/bias removed
- connected day boundaries

MGEX

Simulated (ground test)
- time drift/bias removed
- connected day boundaries

specs

flicker floor
Clock Noise Model: Overlapping Allan Variance from the Ground Test

Ground test of the Galileo H-Maser

Noise model function

\[ f(\tau) = \frac{A^2}{\tau^2} + \frac{B^2}{\tau} + C^2 + D^2\tau \]

- White phase noise: \( 9.8 \times 10^{-13} \) short-term \( \tau < 10 \text{ s} \)
- White frequency noise: \( 5.9 \times 10^{-13} \) short-to-medium term
- Flicker frequency noise: \( 7.9 \times 10^{-16} \) long-term \( \tau > 6 \text{ h} \)
- Frequency drift: \( 1.2 \times 10^{-20}/\text{s} \) (from flight model test)
# Clock Model: MGEX vs. Simulated

## Simulated Galileo H-maser

<table>
<thead>
<tr>
<th>N</th>
<th>0.2 h</th>
<th>0.25 h</th>
<th>0.5 h</th>
<th>1.0 h</th>
<th>1.5 h</th>
<th>6 h</th>
<th>12 h</th>
<th>14 h</th>
<th>24 h</th>
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## MGEX (AIUB)

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## Polynomial removed:
- N=1 Linear model
- N=2 Quadratic model
- N=3
- N=4
- N=5

## Difference MGEX-Simulated

\[
\text{DIFFERENCE} = \sqrt{\text{MGEX}^2 - \text{Simulated H-maser}^2}
\]

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## Ground network noise:

PSD of residual clock parameters 96-106/2013
**$J_2$ Periodic Relativistic Effect**

**$J_2$ Periodic Relativistic Correction**

\[ \Delta t(J_2)_{\text{per}} = -\frac{3}{2} \frac{a_e^2}{a^2 c^2} J_2 \sqrt{GM} \cdot \sin^2 i \sin 2u \]

(Kouba, 2004)

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**Simulated Clock:**

Galileo E11 (Sun Elevations 60°–65°)

**MGEX:**

Amplitude = 18 mm

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PSD of residual clock parameters 96–106/2013

before $J_2$ → after $J_2$
Proper time of the clock:

\[
\frac{d\tau}{dt} = 1 - \frac{1}{c^2} \left[ \frac{v^2}{2} + U_E(x) + V(X_A) - V(X_E) - x_A^i \partial_i V(X_E) \right]
\]

(Petit and Luzum, 2010)

Contribution from Sun and Moon:

\[
V(X_A) - V(X_E) - x_A^i \partial_i V(X_E) = \sum_{A \neq E} GM_A \left[ \frac{1}{r_{A P}} - \frac{1}{r_{A E}} + \frac{x_{AE} x_{EP}}{r_{AE}^3} \right]
\]

(Wolf and Petit, 1995)
Effect of the Earth’s Magnetic Field

Magnetic Sensitivity:
(as measured during ground tests)

\[ <3 \times 10^{-13} / \text{Gauss} \]  

(Boving et al., 2009)

(1 Gauss = 10^{-4} T)

IGRF model:
- total magnetic field variation 300-550 nT
- max. effect < 0.8 mm

Linear model (time drift/bias) removed

Magnetic field intensity along the Galileo E11 Orbit

Max. Accumulated Time due to Magnetic Field Along the Galileo E11 Orbit

assuming the orientation of the Galileo maser cavity along the satellite X-axis (that never faces Sun)
**Thermal Sensitivity**

**Satellite Surface Temperature Along the Orbit:**

Thermal Flux Balance:

\[ q^{\text{in}} = \alpha \cdot q^{\text{Sun}} = q^{\text{out}}_{\text{front}} + q^{\text{out}}_{\text{rear}} \]

**Clock Thermal Sensitivity:**

(as measured during ground tests)

\[ \Delta f/f < < 10^{-15} \]

Max. temperature variation orbit noon-midnight:

\[ \Delta T = 0.07^\circ C - 0.08^\circ C \]

Clock Thermal Sensitivity:

\[ \leq 2 \times 10^{-14}/^\circ C \]  

(Boving et al., 2009)

(Mattioni et al., 2002)

ground platform temperature variations of 5°C

temperature stabilized within \(3 \, \text{m}^\circ C\) (by cavity thermal control)

In addition:

orientation of the Galileo maser cavity is along the satellite +X-axis (that never faces the Sun)

\[ \alpha / \varepsilon \quad (\text{black paint}) = 1.08 \]

\[ \alpha / \varepsilon \quad (\text{kapton}) = 0.63 \]
Conclusions

• **Thermal re-radiation (and thermal inertia) can explain the distinct clock/orbit pattern over a draconic year!**

• SLR bias in Galileo (and GPS) orbits can be explained by orbit shift opposite to the Sun direction due to the thermal re-radiation of the satellite body (SRP is too small for satellite body).

• **Geometrical Mapping of Orbit Perturbations** using onboard GNSS clock is a new technique to monitor orbit errors and was successfully applied to the modelling of thermal re-radiation acceleration (thermal inertia)

• **Galileo clock (MGEX and ground test) is showing smaller standard deviation compared to SLR**

• Simulated Galileo residual clock parameters show a standard deviation of $\sigma=15.5$ mm when time bias and time drift (linear model) is removed over 24 h intervals from the simulated epoch-wise Galileo clock parameters over 10 days, whereas this standard deviation is reduced to $\sigma=11.2$ mm when the linear model is removed every 14 h (orbit period), down to $\sigma=2.7$ mm after time bias and time drift removal on the 1 h.

• The main perturbation affecting the Galileo clock parameters for high Sun elevation ($>60^\circ$) is the periodic relativistic effect due to $J_2$ gravity field coefficient (amplitude of about **18 mm**)

• Accumulated time along the Galileo orbit due to the gravitational potential of Sun and Moon after removing daily time bias and time drift shows distinct 2x per orbit effect below **0.4 mm for the Sun** and **1 mm for the Moon potential**.

• Environmental effects, such as variations in temperature and magnetic field were integrated along the orbit, but did not give a significant impact on the Galileo residual clock parameters. The max. effect of magnetic field is below **0.8 mm** whereas temperature perturbations are well below $1\times10^{-15}$. 
Absolute Code Biases: DCBs Without TEC Maps

Differential Code Bias:

\[ DCB_{P_1, P_2} := AB_1 - AB_2 \]

Absolute Code Bias on \( P_1 \)

Absolute Code Bias on \( P_2 \)

IGS Clock Convention ("Iono-Free Clocks" based on \( P_1 \) and \( P_2 \))

Graphic Linear Combination:

\[ LP_1 = \frac{1}{2}(P_1 + L_1) = \rho + \frac{1}{2}\lambda_1 N_1 + c\delta t - c\delta t^s + \frac{1}{2} AB_1 + \epsilon(LP_1) \]

Iono-Free Linear Combination:

\[ L_{\text{iono-free}} = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = \rho + \lambda N + \frac{1}{2}(\lambda_W - \lambda_N) N_W + c\delta t - c\delta t^s \]

Ambiguity-free condition:

\[ \kappa_1^{af} \lambda_N + \kappa_2^{af} \frac{\lambda_1}{2} := 0 \]

Ambiguity-Free Linear Combination:

\[ AF_1 = \rho + \frac{f_1}{f_1 - f_2} AB_1 - \frac{c \cdot f_2}{(f_1 - f_2)^2} N_W + c\delta t - c\delta t^s \]

Geometry-Free:

\[ AF_1 - P_{\text{iono-free}} = \kappa_1^{af} L_{\text{iono-free}} + \kappa_2^{af} LP_1 - P_{\text{iono-free}} = \frac{\kappa_2^{af}}{2} AB_1 \]
Absolute Code Biases with Third Frequency

\[
L_{\text{iono-free}}^{1,2} = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = \rho + \lambda_N N_1 + \frac{1}{2} \left( \lambda_W - \lambda_N \right) N_W
\]

\[
L_{\text{iono-free}}^{2,5} = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = \rho + \lambda_{N(2,5)} N_1 + \lambda_{N(2,5)} N_W + \frac{1}{2} \left( \lambda_{W(2,5)} - \lambda_{N(2,5)} \right) N_{W(2,5)}
\]

Iono-Free Linear Combinations:

Ambiguity-free condition:
\[
\kappa_1^{af*} \lambda_N + \kappa_2^{af*} \lambda_{N(2,5)} := 0
\]

\[
L_{\text{iono-free}}^{af*} := \kappa_1^{af*} L_{\text{iono-free}}^{1,2} + \kappa_2^{af*} L_{\text{iono-free}}^{2,5}
\]

\[
AF_1 = \rho - \frac{c \cdot f_2}{(f_1 - f_2)^2} N_W - \frac{f_1}{f_1 - f_2} AB_1 + c\delta t - c\delta t^s
\]

Ambiguity-free condition:
\[
- \frac{c \cdot f_2}{(f_1 - f_2)^2} \kappa_1^{af**} + \kappa_2^{af**} \lambda_W := 0
\]

Two-frequency Ambiguity-Free LC (previous slide)

Ambiguity-Free Linear Combination:

Geometry-Free:
\[
AB_1 = \frac{f_1 - f_2}{\kappa_1^{af**} f_1} \left[ (\kappa_1^{af**} AF_1 + \kappa_2^{af**} L_3^{af*} - P_{\text{iono-free}}) - \kappa_2^{af**} \lambda_{W(2,5)} N_{W(2,5)} \right] - \lambda = 3.41 \text{ m}
\]

IGS Clock Convention ("Iono-Free Clocks")
CODE DCBs vs. DCBs Based on the Absolute Code Biases

DCB(P1-P2) vs. DCBs Based on the Absolute Code Biases

- BLOCK IIA
- BLOCK IIR-A
- BLOCK IIR-B
- BLOCK IIR-M
- BLOCK IIF
- Absolute DCBs: 2 Freq.
- Absolute DCBs: 3 Freq.

DCB(P1-P2) - mean(per BLOCK)

- BLOCK IIA
- BLOCK IIR-A
- BLOCK IIR-B
- BLOCK IIR-M
- Two-Freq. BLOCK IIA
- Two-Freq. BLOCK IIR-A
- Two-Freq. BLOCK IIR-B
- Two-Freq. BLOCK IIR-M

λ₁ = 0.43 m
λ₂ = -0.43 m