Estimation of polar motion, polar motion rates, and GNSS orbits in the IGS

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IGS Analysis Center Workshop
Plenary #03 ACC Results and Conventions
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Preamble

- In fall 2012 CODE was obliged to replace its overlapping 3–day solution strategy in favor of a non–overlapping 1–day strategy for its IGS submissions.
- This change had quite a few advantages, but also the problem that the estimated rates of the polar motion components $x$ and $y$ were no longer compatible with the time derivative calculated from the $x$ and $y$ estimates of subsequent days.
- The impact of 1– vs. 3–day orbits on IGS products was studied in detail by:

Contents

- Findings by Lutz el al. (2015)
- Polar motion components $x$ and $y$, angles $x$ and $h$
- The CODE orbits CF2 and CO2 in REPRO2
- The old and new orbit model in 1- and 3-day orbits
- Conclusions/Recommendations
Findings by Lutz et al. (2015)

- The polar motion misclosures of the IGS ACs’ 1–day ERPs at the day boundaries are “of bad quality”. The solution with 3–day orbits (red) does not show problems of this kind.

**Fig. 1** Amplitude spectra of \( y \)-pole coordinate misclosures from REPRO-2 series in 2009–2013 for six 1-day solutions (blue), two 30-hour solutions (green), and a 3-day solution.
Findings by Lutz et al. (2015)

Table 1  Solution series characteristics

<table>
<thead>
<tr>
<th>Solution</th>
<th>Data span</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1/G3</td>
<td>2008–2011</td>
<td>GPS-only, 1- and 3-day</td>
</tr>
<tr>
<td>R1/R3</td>
<td>2008–2011</td>
<td>GLONASS-only, 1- and 3-day</td>
</tr>
<tr>
<td>C1/C3</td>
<td>1994–2013</td>
<td>Combined, 1- and 3-day</td>
</tr>
<tr>
<td>Cx</td>
<td>1994–2013</td>
<td>Combined, with overlapping 3-day orbit arcs but otherwise 1-day products</td>
</tr>
<tr>
<td>Cn</td>
<td>1994–2013</td>
<td>Reconstructed from C3, non-overlapping 3-day solution</td>
</tr>
</tbody>
</table>

- C, Cx, Cn CODE–solutions from REPRO2; C1 and C3 are the classic 1– and 3–day solutions.
Findings by Lutz et al. (2015)

Table 5  Orbit misclosures at day boundaries in the inertial system in mm; period I is based on the years 2002–2008 because of missing GLONASS contributions in the years 2000 and 2001 in the CODE analysis.

<table>
<thead>
<tr>
<th>Sol</th>
<th>GNSS</th>
<th>Period I</th>
<th>Period II</th>
<th>Period III</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>GPS</td>
<td>72</td>
<td>65</td>
<td>68</td>
</tr>
<tr>
<td>C1</td>
<td>GLO</td>
<td>218</td>
<td>115</td>
<td>110</td>
</tr>
<tr>
<td>Cx</td>
<td>GPS</td>
<td>40</td>
<td>27</td>
<td>31</td>
</tr>
<tr>
<td>Cx</td>
<td>GLO</td>
<td>71</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>C3</td>
<td>GPS</td>
<td>41</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>C3</td>
<td>GLO</td>
<td>72</td>
<td>31</td>
<td>31</td>
</tr>
</tbody>
</table>

- C3 and Cx (both based on 3–day orbits) are about a factor of 2 better for GPS, about 3–4 for GLONASS, w.r.t. 1–day solutions.
Findings by Lutz et al. (2015)

- The polar motion rates of 1–day solutions are responsible for bad quality of polar motion misclosures.
- Cn (non-overlapping 3–day) is a good candidate for future CODE IGS contribution.
Polar motion \((x,y)\) and “nutation” \((x,h)\)

- Transformation from Earth–fixed (EF) to inertial (IN) performed by matrix

\[
M(t) = P^T(t) \ N^T(t) \begin{bmatrix} R_3(-\Theta) & R_1(y) & R_2(x) \end{bmatrix}
\]

where \(P, N\) are the precession– and nutation– matrices, \(q\) is sidereal time (including UT1–UTC), \(x\) and \(y\) are the polar motion components. \(R_i(a)\) are rotation matrices about axis \(i\).

- Assuming that \(x\) and \(y\) are small, one may write:

\[
\begin{bmatrix} R_3(-\Theta) & R_1(y) & R_2(x) \end{bmatrix} = \begin{bmatrix} R_1(\eta) & R_2(\xi) & R_3(-\Theta) \end{bmatrix}
\]

where \(x, h\) are rotation angles about the 2nd and 3rd axes of the IN system, respectively.
Polar motion \((x,y)\) and “nutation” \((x,h)\)

- When generating satellite–geodetic solutions including high–resolution ERPs \(x\) and \(y\) one might as well solve for \(x\) and \(h\).
- This theme is treated in detail in the poster by Meindl et al. *Determining sub–daily ERPs using GNSS* in the session *Use of IGS Products*.
- For IGS–like analyses with the two components \((x, y)\) per day&type, one has to study the impact of the linear motion in \((x, y)\) on \((x,h)\) and the approximation of the result by linear functions.
Angles \((x,y)\) and \((x,h)\)

- **Left, red**: effect on angle \(x\) of a constant offset \((x,y) = (1,1)\).
- **Right, red**: effect on angle \(x\) of a constant rate in \((x,y) = (1,1)\).
- **Blue**: linear approximation of \(x\) separately over each day,
- **Green**: piecewise linear approximation (corresponds to CODE 3–day solutions C3,Cn).
CODE orbits CF2 and CO2 in REPRO2

- RMS misclosures in Earth-fixed system, CF2 and CO2 orbits.
- Left: GPS and GLONASS, Right: GPS-only
- Time interval 2002–2013 analyzed (CODE started including GLONASS in 2002)
CODE orbits CF2 and CO2 in REPRO2

- RMS misclosures in Inertial system, CF2 and CO2 orbits.
- Left: GPS and GLONASS, Right: GPS–only
- Time interval 2002–2013 analyzed (CODE started including GLONASS in 2002)
Comparing GNSS orbits referring to the same day

- Conventionally, orbits are compared via 7-parameter Helmert transformations (cf. eg, http://acc.igs.org/).

- The seven parameters are:
  - three offsets,
  - three rotation angles about the coordinate axes,
  - scale parameter

- A 14-parameter Helmert transformation (similarity trafo) was developed, where the rates of the 7 classic parameters may be estimated on top of the classic parameters.

- Subsequently, 6-parameter transformations (the three rotation angles and their rates, only) will be applied to further analyze the difference of orbit systems and the corresponding ERPs.
Comparing GNSS orbits referring to the same day

- Spectra of coordinate differences CF2–CO2 and corresponding pole coordinate in **inertial system**.
- Left: Helmert–trafo, 2\(^{nd}\) axis, right: Helmert–trafo 2\(^{nd}\) axis, rate.
- Orbit differences reflect ERP differences.
Comparing GNSS orbits referring to the same day

- Spectra of coordinate differences CF2–CO2 and corresponding pole differences in Earth–fixed system.
- Left: Helmert–trafo, 2\textsuperscript{nd} axis, right: Helmert–trafo 2\textsuperscript{nd} axis, rate.
- 1– and 3–day orbits are “virtually” the same.
Comparing GNSS orbits referring to the same day

- Spectra of coordinate differences of **CF2** orbits with old and new ECOM and corresponding pole coordinates in **Earth-fixed system**.

- **Left:** Helmert-trafo, 2\textsuperscript{nd} axis, right: Helmert-trafo 2\textsuperscript{nd} axis, rate.
Comparing GNSS orbits referring to the same day

- Spectra of coordinate differences of CO2 orbits with old and new ECOM and corresponding pole coordinates in Earth-fixed system.
- → 3-day solutions are much less sensitive to differences in orbit model …
Conclusions / Recommendations

- Currently, the IGS ACs provide incompatible polar motion coordinates and rates in the sense that the rates are not the first time derivatives of the pole coordinates.

- The current situation is not acceptable (??).

- Solutions to the problem:
  - Go for longer than 1–day arcs – in addition to 1–day solutions
  - Constrain 1–day rates to an IGS/AC–derived a priori pole, which provides consistent coordinates and rates.
Backup slides
Findings by Lutz et al. (2015)

- Other products, e.g., LOD, also improve when replacing the 1–day by the 3–day strategy.
Angles \((x,y)\) and \((x,h)\)

- **Left, red**: effect on angle \(h\) of a constant offset \((x,y) = (1,1)\).
- **Right, red**: effect on angle \(h\) of a constant rate in \((x,y) = (1,1)\).
- **Blue**: linear approximation of \(h\) separately over each day, correspond to IGS 1–day solutions.
- **Green**: piecewise linear approximation (corresponds to CODE 3–day solutions).
Experiments with new ECOM = CFA

Table 7 Quality measures for GLONASS orbits from 1-day solution differences of IGS ACs in the REPRO2 campaign from 2009 to 2013.

<table>
<thead>
<tr>
<th>Solutions</th>
<th>Ref</th>
<th>RMS &lt;sub&gt;b&lt;/sub&gt; (cm)</th>
<th>RMS &lt;sub&gt;n&lt;/sub&gt; (cm)</th>
<th>RMS &lt;sub&gt;e&lt;/sub&gt; (cm)</th>
<th>RMS &lt;sub&gt;H&lt;/sub&gt; (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR2, CF2</td>
<td>EF</td>
<td>11.9</td>
<td>9.1</td>
<td>11.8</td>
<td>9.7</td>
</tr>
<tr>
<td>ES2, CF2</td>
<td>EF</td>
<td>8.6</td>
<td>6.2</td>
<td>8.5</td>
<td>6.5</td>
</tr>
<tr>
<td>GR2, ES2</td>
<td>EF</td>
<td>10.9</td>
<td>8.5</td>
<td>11.3</td>
<td>9.1</td>
</tr>
<tr>
<td>CF2, CFA</td>
<td>EF</td>
<td>8.4</td>
<td>7.7</td>
<td>8.5</td>
<td>7.7</td>
</tr>
<tr>
<td>CO2, CF2</td>
<td>EF</td>
<td>7.5</td>
<td>3.9</td>
<td>7.0</td>
<td>4.0</td>
</tr>
<tr>
<td>CO2, CFA</td>
<td>EF</td>
<td>10.2</td>
<td>8.2</td>
<td>10.1</td>
<td>8.3</td>
</tr>
</tbody>
</table>

- CFA=CF2(new ECOM) can be treated as “just another solution”. The GLONASS–only orbits are analyzed above.
- RMS of misclosures 11.6 cm, 11.0 cm, for CF2, CFA, respectively.
Experiments with new ECOM = CFA

- Helmert angle (rate) about 1st axis (y-pole) w.r.t. CO2
- Left: CF2(Repro2), Right: CFA
Main findings by Lutz el al. (2015)

Table 3 Effect of the arc length on the RMS values of the EOP solutions 2009–2011; units are μas for polar motion, μas/d for rates, μs/d for LOD; offsets removed

<table>
<thead>
<tr>
<th>Periods included</th>
<th>EOP</th>
<th>R1</th>
<th>G1</th>
<th>R3</th>
<th>G3</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
<td>x</td>
<td>238</td>
<td>66</td>
<td>103</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>152</td>
<td>71</td>
<td>95</td>
<td>56</td>
</tr>
<tr>
<td>LOD</td>
<td>35</td>
<td>22</td>
<td>19</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>585</td>
<td>218</td>
<td>136</td>
<td>107</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>990</td>
<td>244</td>
<td>154</td>
<td>113</td>
<td></td>
</tr>
<tr>
<td>&gt;30d</td>
<td>x</td>
<td>151</td>
<td>38</td>
<td>68</td>
<td>42</td>
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<td></td>
<td>y</td>
<td>76</td>
<td>46</td>
<td>65</td>
<td>45</td>
</tr>
<tr>
<td>LOD</td>
<td>18</td>
<td>10</td>
<td>4</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>x</td>
<td>385</td>
<td>134</td>
<td>18</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>y</td>
<td>860</td>
<td>150</td>
<td>26</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

- R1 and R3 are 1- and 3-day GLONASS-only solutions, G1 and G3 GPS-only 1- and 3-day solutions. All ERPs significantly improve from 1 to 3 days. The gain is most significant in the low-pass part (only periods > 30 days included).
Main findings by Lutz el al. (2015)

Table 4  Effect of the arc length on the RMS of the geocenter series for period III (2012–2013); units are mm

<table>
<thead>
<tr>
<th>Periods included</th>
<th>Coord</th>
<th>C1</th>
<th>Cx</th>
<th>C3</th>
</tr>
</thead>
<tbody>
<tr>
<td>all</td>
<td>x</td>
<td>6</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>17</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>&lt;30d</td>
<td>x</td>
<td>2.6</td>
<td>2.5</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>2.6</td>
<td>2.6</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>4.3</td>
<td>3.6</td>
<td>3.1</td>
</tr>
<tr>
<td>&gt;30d</td>
<td>x</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>y</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>17</td>
<td>16</td>
<td>15</td>
</tr>
</tbody>
</table>

- Geocenter coordinates are not much affected by the arc length – apart from RMS of high-pass part, which is, however, a pure filtering effect (three times more data used in C3 than in C1).
Comparing GNSS orbits referring to the same day

- Spectra of $y$-coordinate differences and corresponding ($–x$) coordinate of pole.
- Effects are small: $1\text{mas} \approx 0.128 \text{ mm}$ at maximum at GPS height!
Comparing GNSS orbits referring to the same day

- Spectra of coordinate differences CF2–CO2 and corresponding pole differences in Earth–fixed system.
- Left: Helmert–trafo, 2\textsuperscript{nd} axis, right: Helmert–trafo 2\textsuperscript{nd} axis, rate.
- 1– and 3–day orbits are “virtually” the same.
Comparing GNSS orbits referring to the same day

- Spectra of coordinate differences of CF2 orbits with new ECOM and corresponding pole coordinates in Earth-fixed system.
- Left: Helmert-trafo, 2\textsuperscript{nd} axis, right: Helmert-trafo 2\textsuperscript{nd} axis, rate.
Comparing GNSS orbits referring to the same day

- Rotations of orbits about 2\textsuperscript{nd} Earth–fixed axis and (–x)–coordinate of the pole (offset 200\textmu as).


ES2 and CF2 with old&new ECOM

- Spectra of coordinate differences of CF2 orbits with old&new ECOM and ES2 plus corresponding pole coordinates in Earth-fixed system.
- Left: Helmert-trafo, 2\textsuperscript{nd} axis, right: Helmert-trafo 2\textsuperscript{nd} axis.
- Old ECOM: left, new ECOM: right.
ES2 and CF2 with old&new ECOM

- Spectra of coordinate differences of CF2 orbits with old&new ECOM and ES2 plus corresponding pole coordinates in Earth-fixed system.
- Left: Helmert-trafo, 2\textsuperscript{nd} axis, right: Helmert-trafo 2\textsuperscript{nd} axis, rate.
- Old ECOM: left, new ECOM right