Analyses of GNSS inter-signal delays with geometry-free combinations of GPS and QZSS triple frequency signals

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Abstract
We use three types of geometry-free (GF) code and phase observables formed by triple frequency signals to analyse the uncalibrated code delays (UCD) and uncalibrated phase delays (UPD) and inter-system biases (ISB). In each of the GF phase observables, the lumped phase bias (LPB) that contains a higher-order ionosphere-delay , UPD and UCD, integer ambiguity and ISB terms is generally modelled through a polynomial regression model to account for its time-variations. With a network of multiple receivers, reliable double-differenced (DD) wide-lane and narrow-lane integers (and non-integers) can be obtained by averaging over a long data arc. A network-adjustment procedure is introduced to estimate the UCDs and UPDs, with a boundary condition to overcome the rank deficiency problem and the known DD integers as constraints to remove the effects of ISBs. Numerical results demonstrate how these models and algorithms determine LPBs, resolve the correct DD integers, and estimate satellite- and receiver-specific UCDs and UPDs.

GNSS observation equations and definitions of combinations (6 independent)

Original code and phase equations for frequency f_1, f_2, and f_5 (i=1,2,5):

$$P_i = \rho + \frac{K_1}{f_i^2} + \frac{K_2}{f_i^3} + (b_{i,r} - b_i^s + c_{i,r}^s) + \varepsilon_{Pi} \qquad b_i \equiv b_{i,r} - b_i^s + c_{i,r}^s$$

$$\phi_i = \rho - \frac{K_1}{f_i^2} - \frac{K_2}{2f_i^3} - \lambda_i (N_i + B_i^s - B_{i,r} + C_{i,r}^s) + e_{\phi i} \qquad B_i \equiv B_i^s - B_{i,r} + C_{i,r}^s$$

where: $\rho = |X_r - X^s| + cCLK_r - cCKL^s + m_r^s ZTD_r$

$b_{i,r}$, $B_{i,r}$ receiver-specific hardware biases on code and phase respectively
b_i^s, B_i^s satellite-specific hardware biases on code and phase respectively
$c_{i,r}^s$, $C_{i,r}^s$ Code and phase inter-system biases(ISB) respectively

Linearly combined phase and code equations:

$$P_{(l,m,n)} \equiv \frac{l \cdot f_1 \cdot P_1 + m \cdot f_2 \cdot P_2 + n \cdot f_5 \cdot P_5}{l \cdot f_1 + m \cdot f_2 + n \cdot f_5} \qquad \phi_{(i,j,k)} \equiv \frac{i \cdot f_1 \cdot \phi_1 + j \cdot f_2 \cdot \phi_2 + k \cdot f_5 \cdot \phi_5}{i \cdot f_1 + j \cdot f_2 + k \cdot f_5}$$

Geometry-free /quasi-ionosphere-free observation equations (GF/QIF) (4 independent)

	Unit m	GF(t)	X(t) Lumped Code Bias/Lumped Phase Bias: combined bias term	X(t): 2nd order Ionosphere delay	e(t)
1	1.00	$P_{(1,1,1)} - (\alpha_{11} P_{(1,1,0)} + \alpha_{12} P_{(1,0,1)})$	$b_{(1,1,1)} - (\alpha_{11} b_{(1,1,0)} + \alpha_{12} b_{(1,0,1)})$	$-0.0309 \frac{K_2}{f_1^3}$	$\varepsilon_{P(1,0,0)} - (\alpha_{11} \varepsilon_{P(0,1,0)} + \alpha_{12} \varepsilon_{P(0,0,1)})$
2	WL 0.862	$P_{(1,1,0)} - \varphi_{(1,-1,0)}$	$\{b_{(1,1,0)} + \lambda_{(1,-1,0)} (B_{(1,-1,0)} + N_{(1,-1,0)})\}$	$+0.0226 \frac{K_2}{f_1^3}$	$\varepsilon_{P(1,1,0)} - \varepsilon_{\phi(1,-1,0)}$
3	EWL 5.861	$P_{(0,1,1)} - \phi_{(0,1,-1)}$	$\{b_{(0,1,1)} + \lambda_{(0,1,-1)} (B_{(0,1,-1)} + N_{(0,1,-1)})\}$	$+0.0010 \frac{K_2}{f_1^3}$	$\varepsilon_{P(0,1,1)} - \varepsilon_{\phi(0,1,-1)}$
4	NL 0.107	$P_{(77,-60,0)} - \phi_{(77,-60,0)}$	$\{b_{(77,-60,0)} + \lambda_{(1,1,0)} (B_{(1,0,0)} + N_{(1,0,0)} + \frac{f_2}{f_1 - f_2} B_{(1,-1,0)} + \frac{f_2}{f_1 - f_2} N_{(1,-1,0)}\}$	$-1.0819 \frac{K_2}{f_1^3}$	$\varepsilon_{P(77,-60,0)} - \varepsilon_{\phi(77,-60,0)}$
5	EWL 3.404	$P_{(77,-60,0)} - (\alpha_{21} \phi_{(1,-1,0)} + \alpha_{22} \phi_{(0,1,-1)})$	$b_{(77,-60,0)} + \alpha_{21} \lambda_{(1,-1,0)} (B_{(1,-1,0)} + N_{(1,-1,0)}) + \alpha_{22} \lambda_{(0,1,-1)} (B_{(0,1,-1)} + N_{(0,1,-1)})$	$+0.1380 \frac{K_2}{f_1^3}$	$\varepsilon_{P(77,-60,0)} - (\alpha_{21} \varepsilon_{\phi(1,-1,0)} + \alpha_{22} \varepsilon_{\phi(0,1,-1)})$
6	NL 0.107	$\alpha_{21} \phi_{(1,-1,0)} + \alpha_{22} \phi_{(0,1,-1)} - \phi_{(77,-60,0)}$	$\{-\alpha_{21} \lambda_{(1,-1,0)} (B_{(1,-1,0)} + N_{(1,-1,0)}) - \alpha_{22} \lambda_{(0,1-1)} (B_{(0,1,-1)} + N_{(0,1,-1)}) + \lambda_{(1,1,0)} (B_{(1,0,0)} + N_{(1,0,0)}) + \frac{f_2}{f_1 - f_2} B_{(1,-1,0)} + \frac{f_2}{f_1 - f_2} N_{(1,-1,0)}\}$	$-1.2199 \frac{K_2}{f_1^3}$	$\alpha_{21} \varepsilon_{\phi(1,-1,0)} + \alpha_{22} \varepsilon_{\phi(0,1,-1)} - \varepsilon_{\phi(77,-60,0)}$
Notes:	$\alpha_{11} = -1.7018, \alpha_{12} = 2.7018, \alpha_{21} = \frac{f_1}{f_1 - f_5} = 3.9487, \alpha_{22} = \frac{-f_5}{f_1 - f_5} = -2.9487.$				

Code, Widelane and Narrowlane models for UPD,ISB, and ambiguity terms

Code:

$$LCB_{1,r}^s(t) = UCD_{1,r}(t) - UCD_1^s(t) + ISB_{1,r}^s + e_{1,r}^s(t)$$

Wide-lanes:

$$LPB_{i,r}^s(t) = UPD_{i,r}(t) - UPD_i^s(t) + ISB_{i,r}^s + \lambda_i N_{i,r}^s + e_{i,r}^s(t)$$

$i = 2,3,5$

Narrow-lanes:

$$LPB_{i,r}^s(t) = UPD_{i,r}(t) - UPD_i^s(t) - \theta_i \frac{K_2}{f_1^3} + ISB_{i,r}^s + \lambda_i N_{i,r}^s + e_{i,r}^s(t)$$

$i = 4,6, \quad \theta_4 = 1.0819, \theta_5 = 1.2199$

Motivation:

Processing of multiple GNSS and multiple frequencies require understanding and handling of biases and delays between:

- different GNSS systems;
- between different signals; and
- dependence of signals on satellites and receivers.

Existing approach uses original code and phase observational models, and explicitly describe the observations as the functions of various biases and state parameters, to estimate all together.

- This requires artificial conditions to overcome the linear dependence and rank deficiency.
- It is compute intensive.

We use the geometry-free(GF) and quasi-ionosphere-free(QIF) combinations shown in the above table and estimate UCDs and UPDs independently from the estimation of satellite and receiver states. The overall process is to determine the LPB for each GF/QIF model first, and then separate the UCDs and UPDs from the adjusted LPBs.

Estimation of LPBs for each GF/QIF model:

The estimation of LPBs is completed by two steps.

STEP 1 – Station-based estimation of lumped phase biases (LPB) over a satellite pass or many hours. It is experienced that the 3 and 4-degree polynomial models fit the GF/QIF wide-lane and narrow-lane observables respectively well, as shown in Figure 1 for GMSD to GPS G01 satellite.

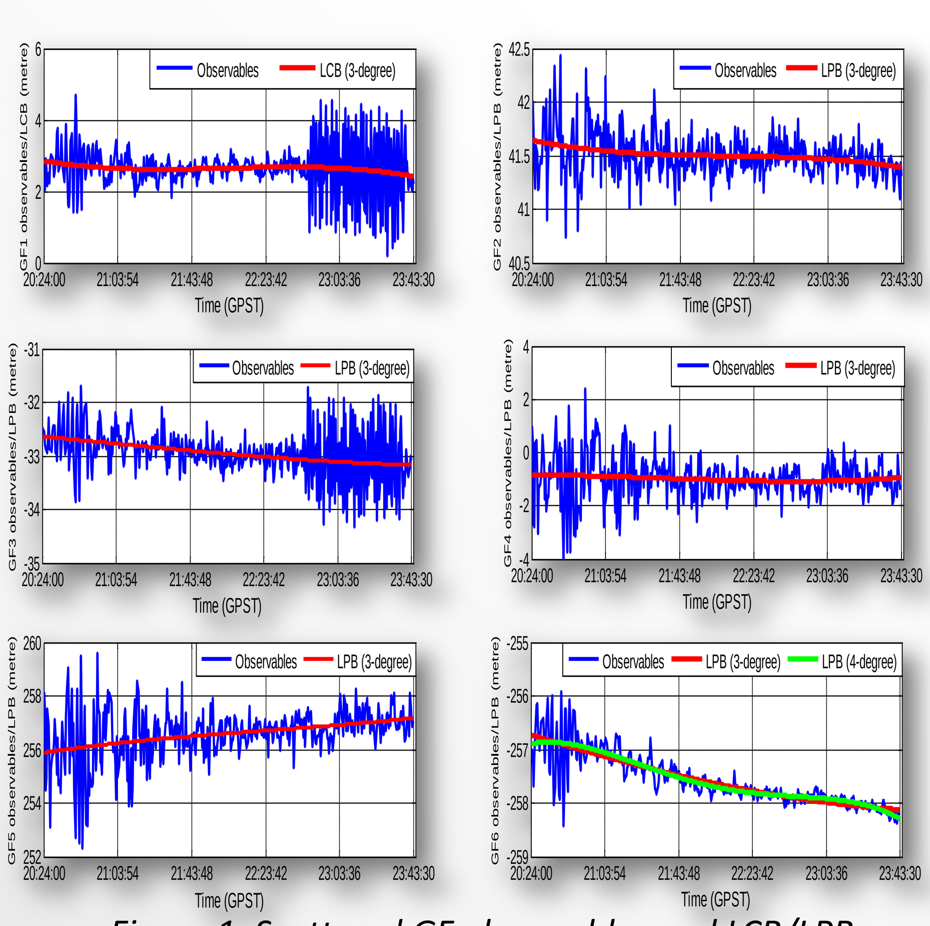


Figure 1. Scattered GF observables and LCB/LPB solutions by polynomial fitting

Table 1. Data sets.

Station ID	Distance to SIN1 (km)	Receiver/ Version	Antenna/ Type	Data type /satellites	Observation period
CUT0 (Australia)	3924	TRIMBLE NETR9/4.80	TRM59800.00 /SCIS	QZSS:C1X L1X C2X L2X C5X L5X GPS: C1C L1C C2X L2X C5X L5X Satellites: GPS G01, QZSS J01	Start Time: 2013-11-23 20:24:00 End Time : 2013-11-23 23:43:30 Epochs: 400 Interval: 30s
GMSD (Japan)	4342				
JFNG (China)	3439		TRM59800.00 /NONE		
SIN1(Singapore)		Trimble NetR9/4.80	LEIAR25.R3 /LEIT		

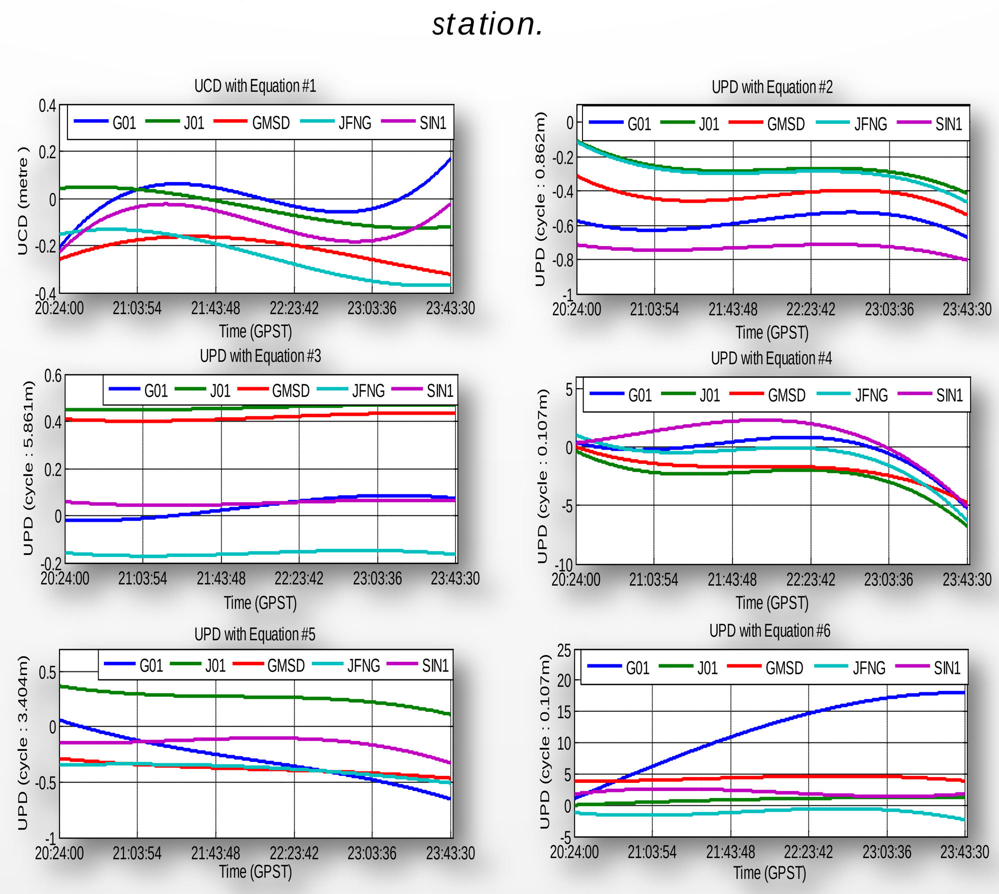
STEP 2 – Network-adjustment of LPBs using the fixed double-differenced (DD) wide-lane and narrow-lanes as constraints which are integers for a singe system and non-integers for different systems. High stability of DD wide-lane and narrow-lane solutions over long distance is critical for this network adjustment. Results are shown in Table 2 and Figures 2a-d. The non-zero values in three DD GF/QIF codes are due to the code ISB between GPS and QZSS. Results show that the wide-lane integers can be correctly fixed within a few minutes of samples. For the two DD narrow-lane integer solutions, much longer cumulative time is required to reach stable and reliable integer solutions.

Network-adjustment for UCD and UPD solutions:

Figure 3 compares the noises of QIF codes and QIF wide-lane after its LPB is estimated and adjusted, showing their consistence and a much lower-noise level of the QIF wide-lane. This is one of the significant advantages of triple frequency.

Based on the network-adjusted LCB/LPB solutions, we estimate satellite-specific UCDs/UPDs for G01 and J01, and station-specific UCDs/UPDs for GMSD, JFNG and SIN1, as shown in Figure 4. The UCD/UPD for CUT0 station is fixed to zero as a reference station. The effect of ISBs on UCD and UPB estimation have been corrected through the DD constraints in the network adjustment process.

Figure 4. UCD and UPD solutions for G01, J01, GMSD, JFNG and SIN1. CUT0 is the reference station.



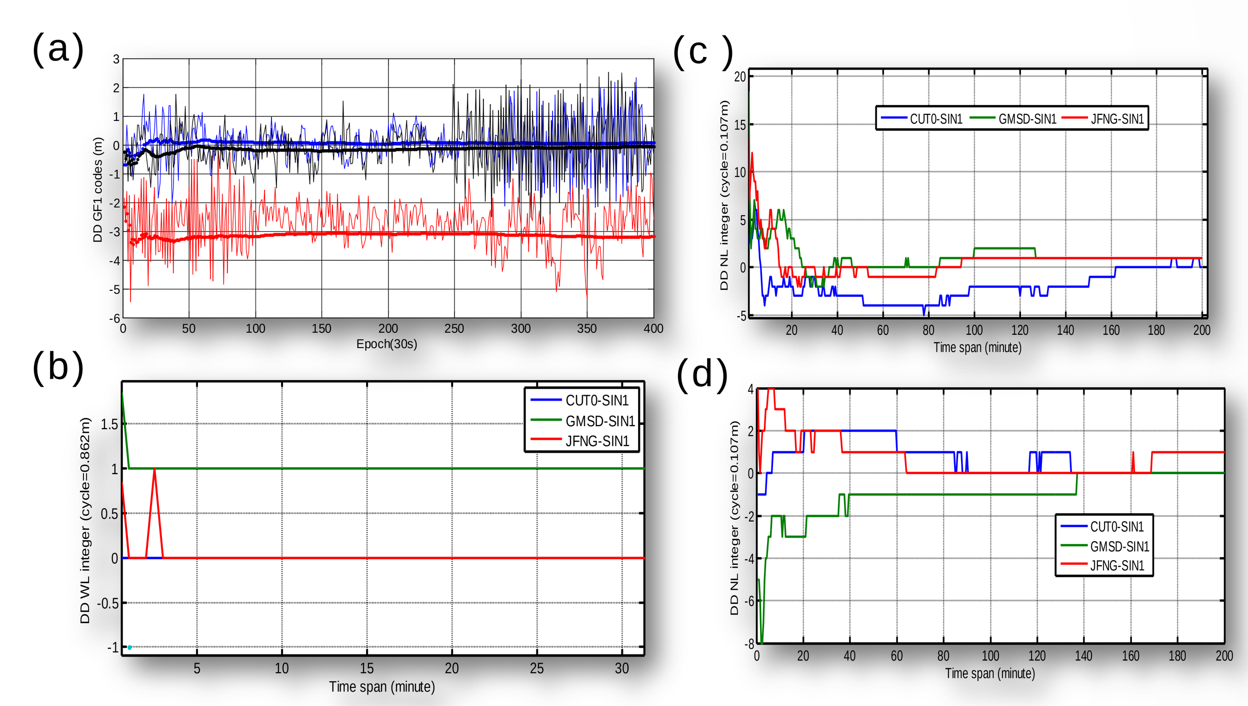
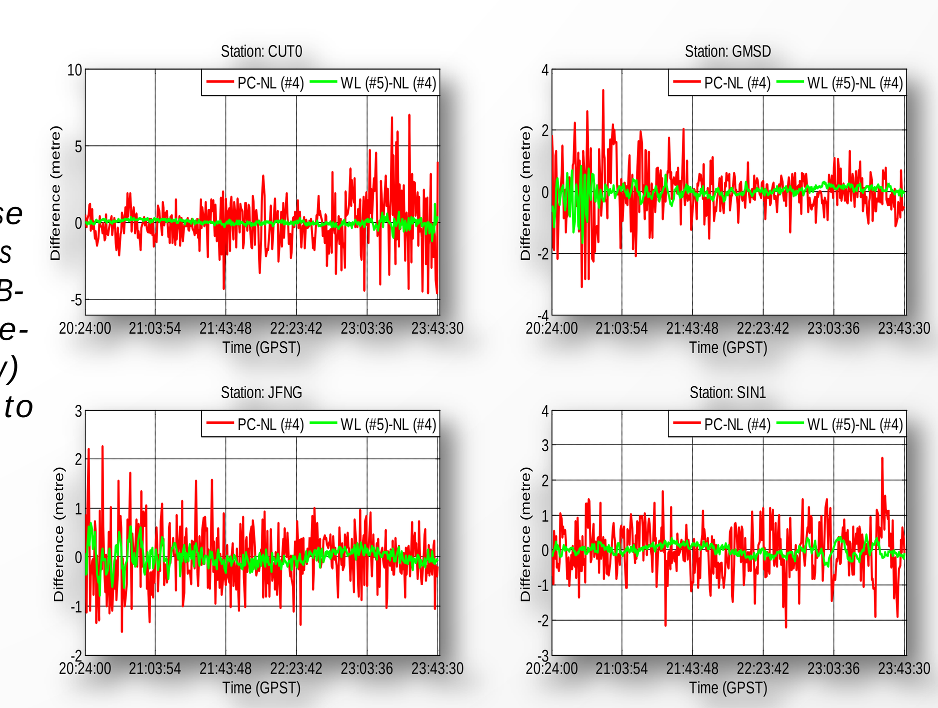


Figure 2. Stability of DD code filtered (a) , wide-lane (b) and narrow-lane (c) and (d) integer solutions

Baseline	GF 1 Unit: 1 m	GF 2 0.862m	GF 3 5.86m	GF 4 0.107m	GF 5 3.404m	GF 6 0.107m
	Mean/std	Mean/std	Mean/std	Mean/std	Mean/std	Mean/std
CUT0-SIN1	-2.740/0.013	0.277/0.004	0.545/0.001	0.360/0.252	0.0385/0.010	0.029/0.035
GMSD-SIN1	0.064/0.001	0.883/0.008	0.026/0.001	0.609/0.180	0.962/0.009	0.323/0.071
JFNG-SIN1	-0.045/0.015	0.100/0.002	0.994/0.000	0.600/0.153	0.733/0.005	0.608/0.035

Table 2. Mean values of DD GF observables over 200 minutes and the standard deviation for the mean values over the last 30 minutes, showing the high stability of the wide-lane mean solutions and lower stability of the narrow-lane solutions

Figure 3. Noise of QIF codes (red) and LPB-fixed QIF wide-lanes (yellow) (Four stations to G01)



Summary:

Use of GF/QIF combination of triple frequency signals allows the receiver and satellite specific hardware-biases to be estimated independently of the estimation of satellite and receiver states. The overall process is to determine the LPB for each GF/QIF model first , and then separate the UCDs and UPDs from the network-adjusted LPBs. The adjusted LPB can be directly used for network-based data processing for estimation of satellite and receiver states. The second-order ionosphere-delay can also be estimated from the GF/QIF equations for narrow-lanes. Numerical results have demonstrated the process and good performance of LPB, UPD and DD GF solutions.