

## ABSTRACT

The goal of this work is to investigate the sensitivity of the estimation of the secular motion of cGNSS stations to several different physical conditions and statistical modeling. We use stations installed in the framework of a collaborative project between JPL and SEGAL that has contributed to densify the global network maintained by JPL. These stations sample the global networks in terms of local environment and spatial distribution on Earth. Our input data are daily positions computed using GIPSY-OASIS (PPP approach) and mapped into the IGS08b reference frame that we use to investigate two major parameters in the estimation of the secular motions: (a) Is indeed the conventional power-law plus white noise model the best stochastic model? Or can different models be applied depending of the local environment of the station, in particular the monument type? In this respect, we apply different noise models using HECTOR (Bos et al., 2013), which can use different noise models and estimate offsets and seasonal signals simultaneously, to investigate how much the choice of a certain stochastic model can change the estimated motion and if correlation exists between the used model and the monument characteristics. (b) How much the existence of data gaps in the time-series can influence the estimation of the secular motion (and also seasonal signals)? Performing several tests with different sub-sets of the same original time-series for several stations permits us to evaluate the maximum acceptable percentage of data gaps (and also their temporal distribution) that does not cause significant variations in the estimated values. This study intends to contribute to improve the estimation of the secular motions by defining best approaches and real constraints on the robustness of the derived motions.

## INTRODUCTION

The stations installed in the framework of the JPL-SEGAL collaboration in the last six years are shown in Figure 1. We just consider in this study the stations installed more than 2 years ago. UFAB is not an exception because it was installed in 2011 - although we just have data for the last 1.6 years.

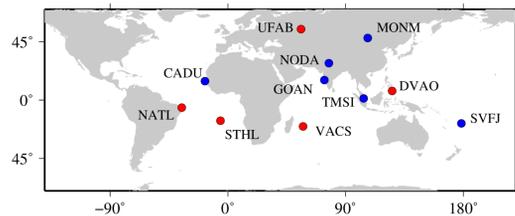


Figure 1 - (Red) stations analyzed in this work; (Blue) other JPL stations installed in collaboration with SEGAL.s

Table 1 - Details of the time-series

Station	Length (years)	gaps (%)	Monument type
UFAB	1.6	47	Metallic Mast Roof Top Local Mount Concrete Pillar
DVAO	2.1	4	Roof Top SEGAL mount Metallic Mast
MONM	3.7	32	Roof Top Local Mount Concrete Pillar
STHL	4.2	3	2m ground SEGAL mount Concrete Pillar
NATL	5.8	47	2m ground SCIGN mount Concrete Pillar
VACS	6.1	4	1.5m ground SEGAL mount

UFAB and MONM were installed in collaboration with local partners who already were running their own systems. A second receiver was installed sharing the existing antenna.

The raw GNSS observations were processed with the GIPSY-OASIS II software (Webb and Zumberge 1995) using the PPP method (Zumberge et al. 1997) to produce daily solutions that were subsequently mapped into ITRF2008 (Altamimi, Métivier, and Collilieux 2012). The linear trends were estimated using the HECTOR software (Bos et al. 2013) that takes into account the temporal correlations that exist within the data. This temporal correlation is represented by a noise model such as power-law noise plus white noise. The parameters of this noise model are estimated in HECTOR using maximum likelihood estimation.

The position time-series for the North component for the stations MONM, VACS and STHL are shown in Figures 2, 3 and 4. MONM has a short data span with significant data gaps. VACS and STHL have records of 4 and 6 years with almost no missing data (and only 1 offset identified at VACS).

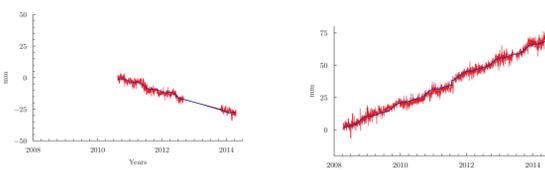


Figure 2 - MONM North component time series

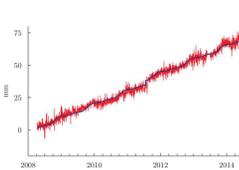


Figure 3- VACS North component time series

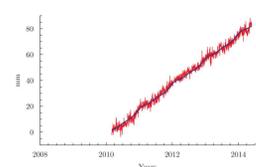


Figure 4- STHL North component time series

The GNSS stations are located in very different parts of the world and we have investigated if they have different noise properties of the data. Figures 5 and 6 show spectral density plots of the residuals (observed position minus estimated linear trend plus a yearly and twice yearly signal and offsets). All stations show white noise at the high frequencies and a power-law behavior at the low frequencies. The slope of this power-law behavior is called the spectral index ( $\kappa$ ). However, from these 2 figures no clear difference in magnitude of the noise levels can be detected.

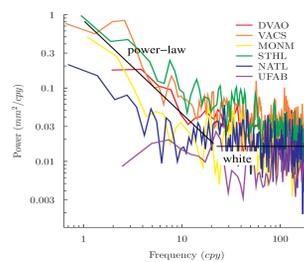


Figure 5 - Power spectra of the residuals for the 6 GNSS stations, East

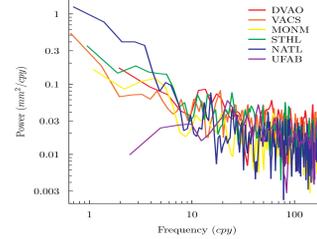


Figure 6 - Power spectra of the residuals for the 6 GNSS stations, North

## INVESTIGATION OF VARIOUS NOISE MODELS

Nowadays it is customary to use a power-law plus white noise model to describe the stochastic properties of the GNSS observations. An alternative would be the Generalized Gauss Markov (GGM) model of Langbein (2008), which is similar to power-law noise but has an extra parameter that enables the noise to flatten at very low frequencies. To investigate its merits, we have also analyzed our data using a GGM plus white noise model. In addition we included a standard fifth-order autoregressive model, AR(5). Examples of the estimated spectra are shown in Figures 7 to 10 for the East and North components, for the stations MONM and VACS.

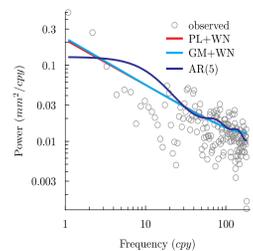


Figure 7 - MONM East component power spectra

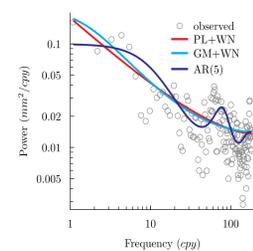


Figure 8- MONM North component power spectra

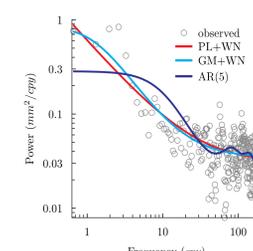


Figure 9 - VACS East component power spectra

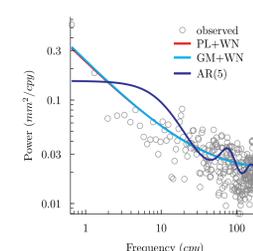


Figure 10 - VACS North component power spectra

Using the Bayesian Information Criterion (Schwarz 1978) we determined that Power-law plus white noise describes the noise slightly better than GGM plus white noise for all stations. Nevertheless, the two models give similar results for the estimated trend uncertainty. The AR(5) noise model however performs significantly worse at all stations.

This is clearly shown in Figures 7 to 10, where we observe that the AR(5) model flattens out at the low frequencies while most GNSS observations continue to show increasing power for decreasing frequency. As a result, AR(5) underestimates the estimated trend error on average by a factor of 2.

## EFFECT OF MISSING DATA ON THE ESTIMATED TREND ERROR

Table 1 shows that several time-series are suffering from missing data. Less data will have an effect on how well we can retrieve the correct stochastic properties of the GNSS data. To investigate this better we created synthetic time-series with power-law plus white noise, similar to those shown in Figure 6 (North component). The spectral index  $\kappa$  was set -0.9 which is close to flicker noise ( $\kappa=-1$ ). The length was chosen to be 1000 days (~3 years) and 3000 days (~8 years). From these time-series various percentages of data were eliminated. These gaps were chosen to be 1) one whole block/segment, randomly positioned or 2) single daily gaps, randomly distributed over the whole the time-series. For each percentage of data gaps, 100 synthetic time-series were generated, both for a single block of missing data and missing data distributed throughout the time-series. The mean estimated spectral indices, the variance of the estimated trend, and the mean estimated trend errors for these synthetic time-series are plotted in Figures 11, 12 and 13 respectively.

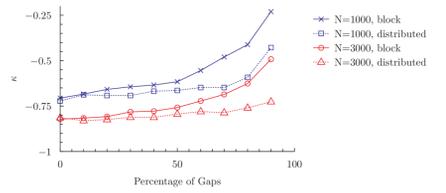


Figure 11- The spectral index  $\kappa$  estimated in time-series with 1000 and 3000 days and various percentages of missing data

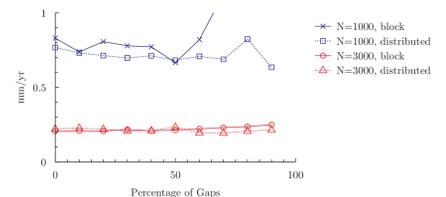


Figure 12- The estimated trend in time-series with 1000 and 3000 days and various percentages of missing data

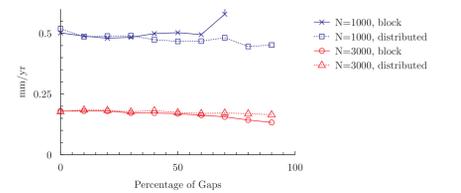


Figure 13- The estimated trend uncertainty in time-series with 1000 and 3000 days and various percentages of missing data

Figure 11 shows that spectral index  $\kappa$  is underestimated in both time-series although the longer time-series is closer to the true value of -0.9. This Figure also shows that for increasing number of missing data, the maximum likelihood estimation results in a more whiter noise than actually exist within the data.

The trend uncertainty is almost entirely determined by the length of the time-series and as Figure 13 shows, almost insensitive to the amount of missing data. In fact, due to the decrease in estimated spectral index, the trend uncertainty decreases slightly with increasing number of missing data. For the short time-series (N=1000) there is so little data left when the percentage of missing data is larger than 60% that the trend error increases rapidly. The same happens for the longer time-series of N=3000 days but only at percentages of 98%. Figures 11 and 13 also shows that it worse to have a single large block with missing data then to have small gaps randomly distributed over the time-series.

## CONCLUSIONS

In this research we investigated 6 GNSS stations that were installed in the framework of RATINHA, a collaborative project between JPL and SEGAL. These stations are located at very different locations, using different monument types.. Nevertheless, we could not find significant difference in the magnitude and behavior of the noise in the observations. Several time-series suffer from to various degree from missing data. To investigate how this affects the estimated trend uncertainty synthetic time-series were generated with various amounts of data gaps. We showed that missing data has little effect on the trend uncertainty but causes the maximum likelihood estimation to favor a pure power-law noise model with higher spectral index above a, more correct, power-law noise model with lower spectral index plus white noise.

## REFERENCES

- Altamimi, Z., Métivier L., and Collilieux, X. 2012. "ITRF2008 Plate Motion Model." *Journal of Geophysical Research (Solid Earth)* 117 (July): 7402. doi:10.1029/2011JB008930.
- Bos, M. S., Fernandes R. M. S., Williams S. D. P., and Bastos, L. 2013. "Fast Error Analysis of Continuous GNSS Observations with Missing Data." *Journal of Geodesy* 87 (4) (December): 351-360. doi:10.1007/s00190-012-0605-0.
- Webb, F. H., and Zumberge, J. F. 1995. "An Introduction to GIPSY/OASIS-II." JPL D-11088. California Institute of Technology, Pasadena, CA.
- Zumberge, J. F., Heflin, M. B., Jefferson D. C., Watkins M. M., and Webb, F. H. 1997. "Precise Point Positioning for the Efficient and Robust Analysis of GPS Data from Large Networks." *J. Geophys. Res.* 102 (B3) (March): 5005-5018. doi:10.1029/96JB03860.